Least squares optimal realisation of autonomous LTI systems is an eigenvalue problem[1]

Prof. Dr. Bart De Moor
Fellow IEEE & SIAM

Bart.demoor@esat.kuleuven.be
www.bartdemoor.be
ESAT-STADIUS, KU Leuven, Belgium

We outline the solution of a long-standing open problem in *system identification and signal processing*, on how to find the *best least squares realisation of an autonomous linear time-invariant (LTI) dynamical system from given data*.

The global optimum is found among all stationary points of a least squares objective function, which we show to correspond to the eigen-tuples of a multi-parameter eigenvalue problem (MEVP). Such an MEVP can be solved by applying Forward (multi-) Shift Recursions to the given set of multivariate polynomial equations, generating so-called block Macaulay matrices, the null space of which can be modelled as the observability matrix of a multi-dimensional shift-invariant linear commutative singular system. The state equations of this system can be found from multi-dimensional realisation theory. From the corresponding eigen-tuples, one can then find the optimal parameters of the best LTI autonomous model.

Our solution methodology uses ingredients from algebraic geometry, operator theory, multi-dimensional system theory and numerical linear algebra, and ultimately requires as basic building blocks only the singular value decomposition and eigen-solvers.

Surprisingly enough, the conclusion is that the globally optimal model in 1D least squares realisation, can be found exactly from multi-dimensional realisation. In addition, we describe several new, previously unknown, properties that characterise the optimal model and its behaviour.

[1] Research funded, among others, by the KU Leuven research fund, the Flanders Science Foundation, an Advanced Grant 'Back to the roots' (No. 885682) from the European Research Council, and by the Flanders AI program.